1/23/20

Last Time: (Some Parameters)

1) (cdf) $F_X(t) = Pr(X \le t)$ (given in tables)

2) (sf) $S_X(t) = Pr(X > t) = 1 - F_X(t)$

3) (pdf) $f_X(t)$ (given in tables)

4) (hazard function) $h_X(t) = \dfrac{f_X(t)}{S_X(t)}$

5) (cumulative hazard function) $H_X(t) = \int_0^t h_X(z)\,dz$

Remark: $S_X(t) = e^{-H_X(t)}$

Examples:

1) Suppose $X \sim Exp(\theta = 1000)$ (i.e. X is a r.v. following an exponential distribution with $\underbrace{\text{mean}}_{\theta} = 1000$)

Determine $f_X(1500)$.

Answer: $f_X(t) = \dfrac{e^{-t/\theta}}{\theta} \Rightarrow f_X(1500) = \dfrac{e^{-1500/1000}}{1000}$

$= .001\, e^{-1.5}$

2) Suppose $f_X(x) = \dfrac{2000000}{(x+1000)^3}$

Determine $S_X(1500)$.

Answer: $X \sim Burr(\alpha = 2, \theta = 1000, \gamma = 1)$

$= 2\text{-Par}(\alpha = 2, \theta = 1000)$

$S_X(x) = \left(\dfrac{\theta}{x+\theta}\right)^{\alpha} \Rightarrow S_X(1500) = \left(\dfrac{1000}{2500}\right)^2$

Example #3) $X \sim$ 2-Pareto$(\alpha = 2, \theta = 5000)$

Determine $H_X(10000)$

Answer:
$$S_X(t) = e^{-H_X(t)}$$

$$S_X(10000) = \left(\frac{5000}{15000}\right)^2 = \frac{1}{9}$$

$$\therefore \frac{1}{9} = e^{-H_X(10000)} \Rightarrow H_X(10000) = -\ln\left(\frac{1}{9}\right) = \ln(9)$$

More Parameters (continued)

6) (Raw) Moments

$$E[X^k] = \int_0^\infty x^k \cdot f_X(x)\,dx \qquad k^{th} \text{ raw moment}$$

(in the tables)

Remark: 1^{st} raw moment

$$\mu = E[X] = \int_0^\infty x \cdot f_X(x)\,dx$$

$u = x \qquad v = -S_X(x)$

$du = dx \qquad dv = f_X(x)\,dx$

$$= \cancel{-x \cdot S_X(x)\Big|_0^\infty} \to 0 + \int_0^\infty S_X(x)\,dx$$

$$\therefore \mu = E[X] = \int_0^\infty x \cdot f_X(x)\,dx = \int_0^\infty S_X(x)\,dx$$

7) $\sigma_X^2 = Var(X) = E[(X-\mu)^2]$

"2nd moment centered about the mean"

From before we saw that

$$\sigma_X^2 = Var(X) = E[X^2] - (E[X])^2$$

Note: $\sigma_X = \sqrt{Var(X)}$ = standard deviation of X

8) Limited Random Variables

Notation: $A \wedge B$ (read "A hat B")

Definition: $A \wedge B = \min(A, B)$

Examples: 1) Let u = maximum covered amount

$$X \wedge u = \min(X, u) = \begin{cases} X & \text{if } X \leq u \\ u & \text{if } X > u \end{cases}$$

$X \wedge u$ = rvr the amount paid by the insurance company per accident

2) Let d = "ordinary" deductible

$$X \wedge d = \min(X, d) = \begin{cases} X & \text{if } X \leq d \\ d & \text{if } X > d \end{cases}$$

$X \wedge d$ = rvr the amount paid by the policy holder per accident

Remark: $E[(X \wedge d)^k] = k^{th}$ limited moment

1st limited moment $= \underbrace{E[X \wedge d]}_{LEV}$ = "Limited Expected Value"

Loss Elimination Ratio = LER = $\dfrac{E[X \wedge d]}{E[X]}$